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QUASI-NORMAL MODES OF TOROIDAL, CYLINDRICAL AND PLANAR BLACK HOLES IN ANTI-DE SITTER SPACETIMES

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Abstract

We study the quasi-normal modes (QNM) of scalar, electromagnetic and gravitational perturbations of black holes in general relativity whose horizons have toroidal, cylindrical or planar topology in an asymptotically anti-de Sitter (AdS) spacetime. The associated quasi-normal frequencies describe the decay in time of the corresponding test field in the vicinities of the black hole. In terms of the AdS/CFT conjecture, the inverse of the frequency is a measure of the dynamical timescale of approach to thermal equilibrium of the corresponding conformal field theory.

1 Introduction

Black holes in anti-de Sitter (AdS) spacetimes in several dimensions have been recently study. One of the reasons for this intense study is the AdS/CFT conjecture which states that there is a correspondence between string theory in AdS spacetime and a conformal field theory (CFT) on the boundary of that space. For instance, type IIB superstring theory on $\mathrm{AdS}_5 \times S^5$ seems to be equivalent to a super Yang-Mills theory in four dimensions [1].

All dimensions up to eleven are of interest in supersting theory, but experiment singles out four dimensions as the most important. In four-dimensional (4D) general relativity, an effective gravity theory in an appropriate string theory limit, the Kerr-Newman family of four-dimensional black holes can be extended to include a negative cosmological constant [2]. The horizon in this family has the topology of a sphere. There are, however, other families of black holes in general relativity with a negative cosmological constant with horizons having topology different from spherical. Here we want to focus on the family of black holes whose horizon has toroidal, cylindrical or planar topology [3, 4, 5] (see [6] for a review). We are going to perturb these black holes with scalar, electromagnetic and gravitational fields.

Perturbations of known solutions are very important to perform in order to study their intrinsic properties, such as the natural frequencies of the perturbations, and to test for the stability of the solutions themselves. For gravitational objects, such as a black hole, the vibrational pattern set by the perturbation obliges the system to emit gravitational waves. Thus, for black holes, the study of perturbations is closely linked to the gravitational wave emission. Due to the dissipative character of the emission of the gravitational waves the vibrational modes do not for a normal set, indeed in the spectrum each frequency is complex whose imaginary part gives the damping timescale. These modes are called quasi-normal modes (QNMs). The QNMs of a black hole appear naturally when one deals with the evolution of some field in the black hole spacetime, and serve as a probe to the dynamics outside its event horizon.

Much work has been done for black holes in asymptotically flat spacetimes (see [7] for a review), the main interest being related to the gravitational waves emitted when these astrophysical objects form. In turn, the recent AdS/CFT correspondence conjecture has attracted much attention to the investigation of QNMs in anti-de Sitter spacetimes. According to it, the black hole corresponds to a thermal state in the conformal field theory, and the decay of the test field in the black hole spacetime, correlates to the decay of the perturbed state in the CFT. The dynamical timescale for the return to thermal equilibrium can be computed using the black hole characteristics plus the AdS/CFT correspondence. Since one can always compactify extra unused dimensions, the study of black holes in any permitted dimension (from 2 to 11) is useful. In 3D QNMs were studied in [8, 9, 10, 11] (see also [12]) for scalar perturbations and in [11] for Maxwell and Weyl perturbations. In 4D QNMs were studied in [8, 13, 14, 15] for Schwarzschild-AdS black holes with scalar perturbations, in [16, 17, 18] for Reissner-Nordström black holes with scalar perturbations, in [19] for Schwarzschild-AdS black holes with Maxwell and gravitational perturbations, and in [20] for topological black holes with scalar perturbations. For higher dimensions, such as 5 and 7D see [9, 13], and for a recent work on super-radiance in the Kerr-Newman-anti-de Sitter geometry see [21].

In this paper we shall study scalar, electromagnetic and gravitational perturbations of the toroidal, cylindrical or planar black holes in an AdS spacetime found in [4]. The motivation to perturb with a scalar field can be seen as follows. If one has, e.g., a 10-dimensional type IIB supergravity, compactified into a (toroidal BH)$_4 \times$ (compact space) the scalar field used to perturb the black hole, can be seen as a type IIB dilaton which couples to a CFT field operator $\mathcal{O}$. Now, the black hole in the bulk corresponds to a thermal sate in the boundary CFT, and thus the bulk scalar perturbation corresponds to a thermal perturbation with nonzero $< \mathcal{O} >$ in the CFT. Similar arguments hold for the electromagnetic perturbations since they can be seen as perturbations for some generic gauge field in 10-dimensional type IIB supergravity. On the other hand, gravitational perturbations are always of importance since they belong to the essence of the spacetime itself.

We will find that the QNM frequencies for scalar perturbations scale with the horizon radius, at least for large black holes. In the case of electromagnetic perturbations of large black holes, the characteristic QNM frequencies have only an imaginary part, and scale with the horizon radius. As for gravitational perturbations, there are two novel features. First, contrary to the asymptotically flat spacetime case, odd and even perturbations no longer have the same spectra, although in certain limits one can still prove that the frequencies are almost the same. The second intriguing result is that, for odd perturbations, there is a mode with a totally different behavior from that found in the scalar and electromagnetic case: in this mode the frequency scales with $\frac{1}{r_+}$, just as in asymptotically flat Schwarzschild spacetime.

2 Scalar, electromagnetic and gravitational perturbations in a toroidal, cylindrical or planar black hole in an AdS background

Throughout this paper, we shall deal with the evolution of some perturbation in a spacetime geometry in general relativity with a background metric given by [5]:

$$ds^2 = f(r)\,dt^2 - f(r)^{-1}dr^2 - r^2\,dz^2 - r^2 d\phi^2 \tag{1}$$

where

$$f(r) = \frac{r^2}{R^2} - \frac{4MR}{r}\,, \tag{2}$$

and R is the AdS lengthscale $R^2 = -\frac{3}{\Lambda}$, Λ being the cosmological constant. The range of the coordinates z and ϕ dictates the topology of the black hole spacetime. For a black hole with toroidal topology, a toroidal black hole, the coordinate z is compactified such that z/R ranges from 0 to 2π, and ϕ ranges from 0 to 2π as well. For the cylindrical black hole, or black string, the coordinate z has range $-\infty < z < \infty$, and $0 \leq \phi < 2\pi$. For the planar black hole, or black membrane, the coordinate ϕ is further decompactified $-\infty < R\phi < \infty$ [5]. We will work with the cylindrical topology but the results are not altered for the other two topologies.

2.1 Scalar field perturbations

For scalar perturbations, we are interested in solutions to the minimally coupled scalar wave equation

$$\Phi^{,\mu}{}_{;\mu} = 0\,, \tag{3}$$

where, a comma stands for ordinary derivative and a semi-colon stands for covariant derivative. We make the following ansatz for the field Φ

$$\Phi = \frac{1}{r} P(r) e^{-i\omega t} e^{ikz} e^{il\phi} \,, \tag{4}$$

where ω, k, and l, are the frequency, the wavenumber and the angular quantum numbers of the perturbation. If one is dealing with the toroidal topology then k should be changed into an angular quantum number $\bar{l}$, $e^{ikz} \to e^{i\bar{l}\frac{z}{R}}$. For the planar topology $e^{il\phi} \to e^{i\bar{k}R\,\phi}$, where $\bar{k}$ is now a continuous wave number.

It is useful to use the tortoise coordinate r_* defined by the equation $dr_* = dr/\left(r^2 - 4MR/r\right)$. With the ansatz (4) and the tortoise coordinate r_*, equation (3) is given by,

$$\frac{d^2 P(r)}{{dr_*}^2} + \left[\omega - V_{\rm scalar}(r)\right] P(r) = 0 \,, \tag{5}$$

where,

$$V_{\rm scalar}(r) = f\left(\frac{l^2}{r^2} + \frac{R^2 k^2}{r^2} + \frac{f'}{r}\right) \,, \tag{6}$$

with $r = r(r_*)$ given implicitly. The rescaling to the the radial coordinate $\hat{r} = \frac{r}{R}$ and to the frequency $\hat{\omega} = \omega R$ is equivalent to take $R = 1$ in (5) and (6), i.e., through this rescaling one measures the frequency and other quantities in terms of the AdS lengthscale R.

2.2 Maxwell field perturbations

We consider the evolution of a Maxwell field in a cylindrical black hole-anti-de Sitter spacetime with a metric (1). The evolution is governed by Maxwell's equations:

$$F^{\mu\nu}{}_{;\nu} = 0 \,, \quad F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \,. \tag{7}$$

One can again separate variables by the ansatz:

$$A_\mu(t,r,\phi,z) = \begin{bmatrix} g^{k\,l}(r) \\ h^{k\,l}(r) \\ k^{k\,l}(r) \\ j^{k\,l}(r) \end{bmatrix} e^{-i\omega t} e^{ikz} e^{il\phi} \,, \tag{8}$$

When we put this expansion into Maxwell's equations (7) we get a 2nd order differential equation for the perturbation:

$$\frac{\partial^2\Psi(r)}{\partial r_*^2}+\left[\omega^2-V_{\rm maxwell}(r)\right]\Psi(r)=0\,, \tag{9}$$

where the wavefunction Ψ is a combination of the functions g^{lm}, h^{lm}, k^{lm} and j^{lm} as appearing in (8), and $\Psi=-i\omega h^{lm}-\frac{dg^{lm}}{dr}$ (see [22] for further details). The potential $V_{\rm maxwell}(r)$ appearing in equation (9) is given by

$$V_{\rm maxwell}(r)=f(r)\left(\frac{l^2}{r^2}+\frac{R^2k^2}{r^2}\right)\,. \tag{10}$$

Again we can take $R=1$ and measure everything in terms of R.

2.3 Gravitational perturbations

In our analysis of gravitational perturbations, we shall adopt a procedure analogous to that of Chandrasekhar [23], generalizing the calculation by the introduction of the cosmological constant $\Lambda=-\frac{3}{R^2}$. The perturbed metric will be taken to be

$$ds^2=e^{2\nu}dt^2-e^{2\psi}(d\phi-wdt-q_2dr-q_3dz)^2-e^{2\mu_2}dr^2-e^{2\mu_3}dz^2\,. \tag{11}$$

where the unperturbed quantities are $e^{2\nu}=\frac{r^2}{R^2}-\frac{4MR}{r}$, $e^{2\psi}=r^2$, $e^{2\mu_2}=(\frac{r^2}{R^2}-\frac{4MR}{r})^{-1}$, $e^{2\mu_3}=\frac{r^2}{R^2}$ and all the other unperturbed quantities are zero. By observing the effect of performing $\phi\to-\phi$, and maintaining the nomenclature of the spherical symmetric case [23], it can be seen that the perturbations fall into two distinct classes: the odd perturbations (also called axial in the spherical case) which are the quantities w, q_2, and q_3, and the even perturbations (also called polar in the spherical case) which are small increments $\delta\nu$, $\delta\mu_2$, $\delta\mu_3$ and $\delta\psi$ of the functions ν, μ_2, μ_3 and ψ, respectively. Note that since z is also an ignorable coordinate, one could interchange ϕ with z in the above argument.

In what follows, we shall limit ourselves to axially symmetric perturbations, i.e., to the case in which the quantities listed above do not depend on ϕ. In this case odd and even perturbations decouple, and it is possible to simplify them considerably.

2.3.1 Odd perturbations

We will deal with odd perturbations first. As we have stated, these are characterized by the non vanishing of w, q_2 and q_3. The equations governing these quantities are the following Einstein's equations with a cosmological constant

$$G_{12} + \frac{3}{R^2} g_{12} = 0\,, \tag{12}$$

$$G_{23} + \frac{3}{R^2} g_{23} = 0\,. \tag{13}$$

The reduction of these two equations to a one dimensional second order differential equation is well known (see [23]), and we only state the results. By defining

$$Z^{-}(r) = r\left(\frac{r^2}{R^2} - \frac{4MR}{r}\right)\left(\frac{dq_2}{dz} - \frac{dq_3}{dr}\right)\,, \tag{14}$$

One can easily check that $Z^{-}(r)$ satisfies

$$\frac{\partial^2 Z^{-}(r)}{\partial r_*^2} + \left[\omega^2 - V_{\rm odd}(r)\right] Z^{-}(r) = 0\,, \tag{15}$$

where $V_{\rm odd}(r)$ appearing in equation (15) is given by

$$V_{\rm odd}(r) = f(r)\left(\frac{R^2k^2}{r^2} - \frac{12MR}{r^3}\right)\,. \tag{16}$$

2.3.2 Even perturbations

Even perturbations are characterized by non-vanishing increments in the metric functions ν, μ_2, μ_3 and ψ. The equations which they obey are obtained by linearizing $G_{01} + \frac{3}{R^2} g_{01}$, $G_{03} + \frac{3}{R^2} g_{03}$, $G_{13} + \frac{3}{R^2} g_{13}$, $G_{11} + \frac{3}{R^2} g_{11}$ and $G_{22} + \frac{3}{R^2} g_{22}$ about their unperturbed values. Making the ansatz

$$\delta\nu = N(r)e^{ikz}\,, \tag{17}$$

$$\delta\mu_2 = L(r)e^{ikz}\,, \tag{18}$$

$$\delta\mu_3 = T(r)e^{ikz}\,, \tag{19}$$

$$\delta\psi = V(r)e^{ikz}\,, \tag{20}$$

we have from $\delta(G_{03}+\frac{3}{R^2}g_{03})=0$ that

$$V(r) = -L(r)\,. \tag{21}$$

Inserting this relation and using (20) in $\delta(G_{01}+\frac{3}{R^2}g_{01})=0$, we have

$$\left(\frac{3}{r}-\frac{f'}{2f}\right)L(r)+\left(\frac{f'}{2f}-\frac{1}{r}\right)T(r)+L(r)_{,r}-T(r)_{,r}=0\,. \tag{22}$$

¿From $\delta(G_{13}+\frac{3}{R^2}g_{13})=0$ we have

$$\left(\frac{1}{r}+\frac{f'}{2f}\right)L(r)+\left(\frac{1}{r}-\frac{f'}{2f}\right)N(r)-N(r)_{,r}+L(r)_{,r}=0\,, \tag{23}$$

and from $\delta(G_{11}+\frac{3}{R^2}g_{11})=0$ we obtain

$$\begin{gathered}-\tfrac{k^2R^2}{fr^2}N(r)+\left(\tfrac{k^2R^2}{fr^2}-\tfrac{\omega^2}{f^2}-\tfrac{6}{f}\right)L(r)+\tfrac{\omega^2}{f^2}T(r)+\\ \left(\tfrac{1}{r}+\tfrac{1}{Rr}\right)N(r)_{,r}+\left(-\tfrac{f'}{2f}-\tfrac{1}{Rr}\right)L(r)_{,r}+\left(\tfrac{f'}{2f}+\tfrac{1}{r}\right)T(r)_{,r}=0\,.\end{gathered} \tag{24}$$

Multiplying equation (23) by $\frac{2}{r}$ and adding (24) we can obtain $N(r)$ and $N(r)_{,r}$ in terms of $L(r)$, $L(r)_{,r}$, $T(r)$ and $T(r)_{,r}$. Using (22) and (23) we can express $L(r)$, $N(r)$, and up to their second derivatives in terms of $T(r)$, $T(r)_{,r}$ and $T(r)_{,rr}$. Finally, we can look for a function

$$Z^+ = a(r)T(r)+b(r)L(r)\,. \tag{25}$$

which satisfies the second order differential equation

$$\frac{\partial^2 Z^+(r)}{\partial r_*^2}+\left[\omega^2-V_{\rm even}(r)\right]Z^+(r)=0\,, \tag{26}$$

Substituting (25) into (26) and expressing $L(r)$ and its derivatives in terms of $T(r)$ and its derivatives, we obtain an equation in $T(r)$, $T(r)_{,r}$ and $T(r)_{,rr}$ whose coefficients must vanish identically. If we now demand that $a(r)$ and $b(r)$ do not depend on the frequency ω we find

$$a(r) = \frac{r}{12Mr+k^2r^2}\,, \tag{27}$$

$$b(r) = \frac{6M+k^2r}{72M^2+6k^2Mr}\,, \tag{28}$$

and the potential $V_{\rm even}(r)$ in (26) is

$$V_{\rm even}(r) = f(r)\left[\frac{576M^3 + 12k^4Mr^2 + k^6r^3 + 144M^2r(k^2+2r^2)}{r^3(12M+k^2r)^2}\right] . \qquad (29)$$

As a final remark concerning the wave equations obeyed by odd and even gravitational perturbations, we note that it can easily be checked that the two potentials can be expressed in the form

$$V_{\substack{\rm odd \\ \rm even}} = W^2 \pm \frac{dW}{dr_*} + \beta\,, \qquad (30)$$

$$W = \frac{96M^2(k^2+3r^2)}{2k^2r^2(12M+k^2r)} + j\,, \qquad (31)$$

where $j = -\frac{k^6+288M^2}{24k^2M}$, and $\beta = -\frac{k^8}{576M^2}$. It is worth of notice that the two potentials can be written in such a simple form (potentials related in this manner are sometimes called superpartner potentials [24]), a fact which seems to have been discovered by Chandrasekhar [23].

3 Quasinormal modes and some of its properties

3.1 Boundary conditions

To solve (5), (9), (15) and (26) one must specify boundary conditions, a non-trivial task in AdS spacetimes. Consider first the case of a Schwarzschild black hole in an asymptotically flat spacetime (see [7]). Since the potential now vanishes at both infinity and the horizon, two independent solutions near these points are $\Psi_1 \sim e^{-i\omega r_*}$ and $\Psi_2 \sim e^{i\omega r_*}$, where the r_* coordinate now ranges from $-\infty$ to ∞. Quasinormal modes are defined by the condition that at the horizon there are only ingoing waves, $\Psi_{\rm hor} \sim e^{-i\omega r_*}$. Furthermore, one wishes to have nothing coming in from infinity (where the potential now vanishes), so one wants a purely outgoing wave at infinity, $\Psi_{\rm infinity} \sim e^{i\omega r_*}$. Clearly, only a discrete set of frequencies ω meet these requirements.

Consider now our asymptotically AdS spacetime. The first boundary condition stands as it is, so we want that near the horizon $\Psi_{\rm hor} \sim e^{-i\omega r_*}$.

However r_* has a finite range, so the second boundary condition needs to be changed. There have been a number of papers on which boundary conditions to impose at infinity in AdS spacetimes ([25]-[27]). We shall require energy conservation and adopt reflective boundary conditions at infinity [25] which means that the wavefunction is zero at infinity (see however [28]).

3.2 Numerical calculation of the QNM frequencies

To find the frequencies ω that satisfy the previously stated boundary conditions we first change wavefunction to $\phi = e^{i\omega r_*}Z$ (where, $Z = P\,, \Psi\,, Z^+\,, Z^-$). The wave equation then transforms into

$$f(r)\frac{\partial^2\phi}{\partial r^2} + (f' - 2i\omega)\,\frac{\partial\phi}{\partial r} - \frac{V}{f}\phi = 0\,, \tag{32}$$

where $f(r) = (r^2 + 1 - \frac{4M}{r})$.

We now note that (32) has only regular singularities in the range of interest. It has therefore, by Fuchs theorem, a polynomial solution. To deal with the point at infinity, we first change the independent variable to $x = \frac{1}{r}$. Now we can use Fröbenius method by looking for an indicial equation (for further details see [13]), and force it to obey the boundary condition at the horizon ($x = \frac{1}{r_+} = h$). We get

$$Z(x) = \sum_{n=0}^{\infty} a_{n(\omega)}(x-h)^n\,, \tag{33}$$

where $a_{n(\omega)}$ is a function of the frequency. If we put (33) into (32) and use the boundary condition $Z = 0$ at infinity ($x = 0$) we get:

$$\sum_{n=0}^{\infty} a_{n(\omega)}(-h)^n = 0 \tag{34}$$

Our problem is reduced to that of finding a numerical solution of the polynomial equation(34). The numerical roots for ω of equation(34) can be evaluated, resorting to numerical computation. Obviously, one cannot determine the full sum in expression(34), so we have to determine a partial sum from 0 to N, say and find the roots ω of the resulting polynomial expression. We then move onto the next term $N+1$ and determine the roots. If the method is reliable, the roots should converge. We stop our search one we have a 3 decimal digit precision.

As we will see there are frequencies with a vanishing real part, which makes it possible to use an approximation, due to Liu, to these highly damped modes [29, 30]. Although the method was originally developed for the asymptotically flat space, it is quite straightforward to apply it to our case. There is therefore a way to test our results. Unfortunately, this method relies heavily on having not only a pure imaginary frequency but also a frequency with a large imaginary part, so as we shall see it will only work for electromagnetic perturbations. We have computed the lowest frequencies for some values of the horizon radius r_+, and l. The frequency is written as $\omega = \omega_r + i\omega_i$, where ω_r is the real part of the frequency and ω_i is its imaginary part. We present the results in tables 1-4.

3.2.1 Scalar:

	Numerical	
r_+	$-\omega_i$	ω_r
0.1	0.266	0.185
1	2.664	1.849
5	13.319	9.247
10	26.638	18.494
50	133.192	92.471
100	266.373	184.942

Table 1. Lowest QNM of scalar perturbations for $l = 0$ and $k = 0$.

In tables 1 and 2 we list the numerical values of the lowest quasinormal frequencies for $l = 1$,$l = 2$ and for selected values of r_+ . For frequencies with no real part, we list the values obtained in the “highly damped approximation”[29, 30].

3.2.2 Electromagnetic:

	Numerical		Highly Damped	
r_+	$-\omega_i$	ω_r	$-\omega_i$	ω_r
0.1	0.104	1.033	–	–
1	1.709	1.336	–	–
5	7.982	~ 0	7.500	~ 0
10	15.220	~ 0	15.000	~ 0
50	75.043	~ 0	75.000	~ 0
100	150.021	~ 0	150.000	~ 0

Table 2. Lowest QNM of electromagnetic perturbations for $l = 1$ and $k = 0$.

As one can see, the imaginary part of the frequency, which determines how damped the mode is, and which according to the AdS/CFT conjecture is a measure of the characteristic time $\tau = \frac{1}{\omega_i}$ of approach to thermal equilibrium, scales linearly (for large black holes) with the horizon radius supporting the arguments given in [13]. Moreover, the frequencies don't seem to depend on the angular quantum number l (we have performed calculations for higher values of l), and are in the electromagnetic case in excellent agreement with the analytical approximation for strongly damped modes.

3.2.3 Gravitational:

The numerical calculation of the quasinormal frequencies for gravitational perturbations proceeds as outlined previously (the associated differential equation has only regular singularities, so it is possible to use an expansion such as (33)). In tables 3 and 4 we show the two lowest lying QNM frequencies for $l = 2$ and $l = 3$ gravitational perturbations.

We first note that there is clearly a distinction between odd and even perturbations: they no longer have the same spectra, contrary to the asymptotically flat space case (see [31]). This problem was studied in some detail by Cardoso and Lemos [19] who showed that it is connected with the behavior of W (see equation (31)) at infinity.

odd (axial) modes:

	lowest QNM		second lowest QNM	
r_+	$-\omega_i$	ω_r	$-\omega_i$	ω_r
1	2.646	~ 0	2.047	2.216
5	0.2703	~ 0	13.288	9.355
10	0.13378	~ 0	26.623	18.549
50	0.02667	~ 0	133.189	92.482
100	0.0134	~ 0	266.384	184.948

Table 3. Lowest QNM of gravitational odd perturbations for $k = 2$

even (polar) modes:

	lowest QNM, $k = 2$	
r_+	$-\omega_i$	ω_r
1	1.552	2.305
5	12.633	9.624
10	26.296	18.696
50	133.124	92.512
100	266.351	184.963

Table 4. Lowest QNM of gravitational even perturbations for $k = 2$.

We also remark that in electromagnetic and scalar perturbations the frequency scales with r_+ (for large black holes at least). Since the temperature scales also with r_+ in the large black hole regime, this means that the frequency scales with the temperature. Thus, in the dual CFT the approach to thermal equilibrium is faster for higher temperatures. This is a totally different behavior from that of asymptotically flat space, in which the frequency scales with $\frac{1}{r_+}$. However, for odd gravitational modes there is one that scales with $\frac{1}{r_+}$. This is a reflection of the different behavior of the potential $V_{\rm odd}$ for odd perturbations. We finally point out the remarkable resemblance of the values in tables 1, 3 and 4 for scalar and gravitational perturbations, even though the potentials are so different. We have performed calculations for higher values of the angular quantum number l, and found that the QNM frequencies are indeed very similar throughout all values of l.

4 Conclusions

We have computed the scalar, electromagnetic and gravitational QNM frequencies of the toroidal, cylindrical or planar black hole in four dimensions. These modes dictate the late time behaviour of a minimally coupled scalar, electromagnetic field and of small gravitational perturbations, respectively. The main conclusion to be drawn from this work is that these black holes are stable with respect to small perturbations. In fact, as one can see, the frequencies all have a negative imaginary part, which means that these perturbations will decay exponentially with time. For odd gravitational perturbations in the large black hole regime, the imaginary part of the frequency goes to zero scaling with $\frac{1}{r_+}$, just as in asymptotically flat space. In terms of the AdS/CFT correspondence, this implies that the greater the mass, the more time it takes to approach equilibrium, a somewhat puzzling result. Apart from this interesting result, the frequencies all scale with the horizon radius, at least in the large black hole regime, supporting the arguments given in [13]. The QNM for toroidal, cylindrical or planar black holes (in anti-de Sitter space) are quite similar to those of the Scharzschild-anti-de Sitter black hole [13, 19].

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References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 253 (1998).

[2] B. Carter, "General theory of stationary black hole states", in *Black Holes*, eds B. DeWitt, C. DeWitt, (Gordon Breach, New York 1973).

[3] J. P. S. Lemos, Phys. Lett. B **353**, 46 (1995).

[4] J. P. S. Lemos, Class. Quantum Gravity **12**, 1081 (1995).

[5] J. P. S. Lemos, V. T. Zanchin, Phys. Rev. D **54**, 3840 (1996).

[6] J.P.S. Lemos, "Toroidal and cylindrical black holes in anti-de Sitter spacetimes and their properties", in *New trends in Astronomy and Astrophysics*, Proceedings of the X$^{\rm th}$ Encontro Nacional de Astronomia e Astrofísica, eds., J. P. S. Lemos, A. Mourão, L. Teodoro, R. Ugoccioni, (World Scientific 2001), gr-qc/0011092.

[7] K. D. Kokkotas, and B. G. Schmidt, Living Rev. Rel. **2**, 2 (1999).

[8] S. F. J. Chan, and R. B. Mann, Phys. Rev. **D55**, 7546 (1997).

[9] T. R. Govindarajan, and V. Suneeta, Class. Quantum Grav. **18**, 265 (2001).

[10] D. Birmingham, hep-th/0101194.

[11] V. Cardoso, J.P.S. Lemos, Phys. Rev. D **63**, 124015 (2001).

[12] I. Ichinose, and K. Satoh, Nucl. Phys. **B447**, 340 (1995).

[13] G. T. Horowitz, V. Hubeny, *Phys. Rev.* **D62**, 024027 (2000).

[14] G. T. Horowitz, *Class. Quantum Grav.* **D17**, 1107 (2000).

[15] J. Zhu, B. Wang, E. Abdalla, hep-th/0101133.

[16] B. Wang, C. M. Mendes, E. Abdalla, *Phys. Lett.* **B481**, 79 (2000).

[17] B. Wang, C. M. Mendes, E. Abdalla, *Phys. Rev.* **D63**, 084001 (2001).

[18] W. T. Kim, J. J. Oh, hep-th/0105112.

[19] V. Cardoso, J.P.S. Lemos, Phys. Rev. D, in press (2001); gr-qc/0105103.

[20] J.S.F. Chan and R.B. Mann, Phys. Rev. D59, 064019 (1999).

[21] E. Winstanley, gr-qc/0106032.

[22] R. Ruffini, in *Black Holes: les Astres Occlus*, (Gordon and Breach Science Publishers, 1973).

[23] S. Chandrasekhar, in *The Mathematical Theory of Black Holes*, (Oxford University Press, New York, 1983).

[24] F. Cooper, A. Khare and U. Sukhatme, Phys. Rep. **251**, 267 (1995) .

[25] S. J. Avis, C. J. Isham and D. Storey, Phys. Rev. **D18**, 3565 (1978).

[26] P. Breitenlohner and D. Z. Freedman, *Phys. Lett.* **B115**, (1982) 197.

[27] C. P. Burgess and C. A. Lutken, Phys. Lett. **B153**, 137 (1985).

[28] A. Dasgupta, Phys. Lett. **B445**, 279 (1999).

[29] H. Liu, *Class. Quantum Grav.* **12**, 543 (1995).

[30] H. Liu, and B. Mashhoon, *Class. Quantum Grav.* **13**, 233 (1996).

[31] S. Chandrasekhar, S. Detweiler, Proc. R. Soc. London, Ser. A **344**, 441 (1975).